MEASI
Institute of Management

SIX SIGMA STUDY NOTES

OVERVIEW OF SIX SIGMA

Underlying concept of variation, the relationships to related Quality Management approaches, basic Six Sigma tools, international ISO standards for Six Sigma, and the nature of Six Sigma improvement projects, DMAIC Methodology Overview, Financial Benefits of Six Sigma, The Impact of Six Sigma on the Organization. Project Definition: Project Charter, developing a Business Case, chartering a Team,

Defining Roles and Responsibilities, Gathering Voice of the Customer, Support for Project, Translating Customer Needs into Specific Requirements (CTQs), and SIPOC Diagram.

MEASURE

Process Mapping (As-Is Process), Data Attributes (Continuous Versus Discrete), Measurement System Analysis, Data Collection Techniques, Data Collection Plan, Understanding Variation, Measuring Process Capability, Calculating Process Sigma Level, Visually Displaying Baseline Performance. Statistics, Probability and Probability Distribution, Measurement System Analysis, Process Performance Analysis.

ANALYZE

Visually Displaying Data (Histogram, Run Chart, Pareto Chart, Scatter Diagram), Detailed (Lower Level) Process Mapping of Critical Areas, Value-Added Analysis, Cause and Effect Analysis (a.k.a. Fishbone, Ishikawa), Affinity Diagram, Data Segmentation and Stratification, Verification of Root Causes, Determining Opportunity (Defects and Financial) for Improvement. Data Analysis, Test of Hypothesis, Design of Experiment, FMEA, and QFD.

IMPROVE

Design of Experiment, FEMA, and QFD, Brainstorming, Multi-Voting, Quality Function Deployment (House of Quality), Selecting a Solution, Failure Modes and Effects Analysis (FMEA), Poka Yoke (Mistake Proofing Your New Process), Piloting Your Solution, Implementation Planning. Control: Assessing the Results of Process

Improvement, Statistical Process Control (SPC) Overview, developing a Process Control Plan, and Documenting the Process.

CONTROL

Statistical Process Control, Operating Characteristic (OC) Curve for Variable Control, Attribute Control charts, Minitab Application, Acceptance Sampling, Design for Six Sigma (DFSS), DMADV, DMADOV, and DFX

Underlying Concept of Variation

- **Variation** is the enemy of quality - it represents inconsistency in processes and outputs

- Six Sigma aims to reduce process variation to achieve near-perfect results (3.4 defects per million opportunities)

- Types of variation:

  - **Common cause variation**: Natural, inherent to the process

  - **Special cause variation**: Unusual, specific events that can be identified and eliminated

Sigma Levels

- 1 Sigma = 691,462 DPMO (defects per million opportunities)

- 3 Sigma = 66,807 DPMO

- 6 Sigma = 3.4 DPMO (99.99966% perfection)

Relationships to Quality Management Approaches

- **Total Quality Management (TQM)**: Six Sigma builds on TQM principles with more structured methodology

- **Lean Manufacturing**: Often combined as "Lean Six Sigma" - Lean eliminates waste, Six Sigma reduces variation

- **ISO 9000**: Six Sigma can support ISO certification efforts

- **Kaizen**: Continuous improvement philosophy incorporated into Six Sigma culture

Basic Six Sigma Tools

- **Process Mapping**: Visual representation of workflow

- **Fishbone Diagram (Ishikawa)**: Cause-and-effect analysis

- **Pareto Chart**: 80/20 rule visualization

- **Control Charts**: Monitor process stability over time

- **Histogram**: Display data distribution

- **Scatter Diagram**: Show relationships between variables

- **Check Sheets**: Data collection forms

International ISO Standards for Six Sigma

- **ISO 13053**: Quantitative methods in process improvement - Six Sigma

  - Part 1: DMAIC methodology

  - Part 2: Tools and techniques

- **ISO 18404**: Guidelines on competence for Six Sigma practitioners

- These standards provide framework for implementing Six Sigma globally

Nature of Six Sigma Improvement Projects

- **Data-driven**: Decisions based on facts, not opinions

- **Project-based**: Focused initiatives with clear start and end

- **Cross-functional**: Teams from different departments

- **Customer-focused**: Improvements tied to customer requirements

- **Financial impact**: Projects must show measurable ROI

2. DMAIC Methodology Overview

DMAIC is the core problem-solving framework in Six Sigma:

Define

- Identify the problem and project goals

- Define customer requirements

- Create project charter

Measure

- Determine what to measure

- Collect baseline data

- Assess current process capability

Analyze

- Identify root causes of defects

- Validate causes with data

- Determine process drivers

Improve

- Develop solutions to address root causes

- Test improvements on small scale

- Implement full-scale solutions

Control

- Establish controls to sustain improvements

- Monitor ongoing performance

- Document the improved process

3. Financial Benefits of Six Sigma

Direct Benefits

- **Reduced defects**: Lower rework and scrap costs

- **Improved efficiency**: Less waste, faster cycle times

- **Cost savings**: Typically $150K-$250K per completed project

- **Increased capacity**: Same resources produce more output

Indirect Benefits

- **Customer satisfaction**: Better quality leads to loyalty

- **Market share growth**: Competitive advantage

- **Employee morale**: Pride in quality work

- **Risk reduction**: Fewer errors and compliance issues

ROI Calculation

- Most organizations see 3:1 to 6:1 return on Six Sigma investment

- Typical project duration: 3-6 months

- Black Belt expected to complete 4-6 projects annually

4. Impact of Six Sigma on the Organization

Cultural Changes

- Data-driven decision making becomes the norm

- Focus shifts from firefighting to prevention

- Continuous improvement mindset

- Increased collaboration across departments

Organizational Structure

- **Champions**: Senior leaders who sponsor projects

- **Master Black Belts**: Expert practitioners and coaches

- **Black Belts**: Full-time project leaders

- **Green Belts**: Part-time project leaders

- **Yellow Belts**: Team members with basic knowledge

Strategic Alignment

- Projects tied to business objectives

- Resources allocated to highest-impact initiatives

- Consistent methodology across the organization

5. Project Definition

Project Charter

A formal document that authorizes the project, including:

- **Problem statement**: Clear description of the issue

- **Goal statement**: Specific, measurable objectives

- **Scope**: What's included and excluded

- **Timeline**: Start and end dates, milestones

- **Team members**: Roles and time commitments

- **Estimated benefits**: Financial and non-financial

Developing a Business Case

Answer these questions:

- Why is this project important?

- What's the cost of doing nothing?

- What benefits will the project deliver?

- How does it align with strategy?

- What resources are required?

Chartering a Team

- **Size**: Typically 4-8 members

- **Composition**: Cross-functional representation

- **Skills**: Mix of process knowledge and analytical ability

- **Commitment**: Time allocation agreement

- **Authority**: Decision-making boundaries

Defining Roles and Responsibilities

Project Sponsor/Champion:

- Provides resources and removes barriers

- Reviews progress regularly

- Makes high-level decisions

Black Belt/Project Leader:

- Leads the project day-to-day

- Applies Six Sigma methodology

- Facilitates team meetings

- Reports progress

Team Members:

- Contribute subject matter expertise

- Collect and analyze data

- Implement solutions

Process Owner:

- Has authority over the process

- Sustains improvements after project closes

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6. Gathering Voice of the Customer (VOC)

Methods to Capture VOC

- **Surveys**: Structured questionnaires

- **Interviews**: One-on-one conversations

- **Focus groups**: Facilitated discussions

- **Observation**: Watch customers use product/service

- **Complaint data**: Analyze existing feedback

- **Social media**: Monitor online conversations

Types of Customer Requirements

- **Spoken**: Explicitly stated needs

- **Unspoken**: Assumed expectations (basic requirements)

- **Exciting**: Unexpected features that delight

VOC Analysis

- Categorize feedback into themes

- Prioritize based on importance and frequency

- Translate qualitative data into measurable requirements

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7. Support for Project

Stakeholder Management

- **Identify stakeholders**: Who will be affected?

- **Assess influence**: Who has power to help or hinder?

- **Communication plan**: Keep stakeholders informed

- **Address concerns**: Proactively manage resistance

Resource Requirements

- **Time**: Team member availability

- **Budget**: For data collection, tools, implementation

- **Expertise**: Access to technical specialists if needed

- **Technology**: Software, equipment, systems access

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8. Translating Customer Needs into CTQs

CTQ (Critical to Quality) Characteristics

CTQs are measurable performance standards derived from customer requirements.

Process:

1. Start with customer need (qualitative)

2. Break down into specific requirements

3. Define measurable characteristics

Example:

- Customer need: "Fast delivery"

- CTQ: "Order delivered within 48 hours of placement"

- Measure: Delivery time in hours

CTQ Tree

A visual tool that translates needs:

```

Customer Need → Driver → CTQ (Measurable)
```

Example:

"Good Service" →

  - Fast → Wait time < 5 minutes

  - Friendly → Customer satisfaction score > 4.5/5

  - Accurate → Order accuracy > 99%

```

### Characteristics of Good CTQs

- **Specific**: Clearly defined

- **Measurable**: Can be quantified

- **Actionable**: Team can influence it

- **Relevant**: Tied to customer satisfaction

- **Time-bound**: Target timeframe specified

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## 9. SIPOC Diagram

### What is SIPOC?

A high-level process map showing:

- **S**uppliers: Who provides inputs

- **I**nputs: What goes into the process

- **P**rocess: Main steps (5-7 high-level)
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- **O**utputs: What the process produces

- **C**ustomers: Who receives outputs

When to Use SIPOC

- At the start of a project (Define phase)

- To establish process boundaries

- To ensure team has common understanding

- To identify key stakeholders

Creating a SIPOC

1. Start with the Process (P) - list 5-7 major steps

2. Identify Outputs (O) - what does the process produce?

3. Identify Customers (C) - who receives the outputs?

4. Identify Inputs (I) - what's needed to run the process?

5. Identify Suppliers (S) - who provides the inputs?

Example SIPOC (Order Fulfillment):

MEASURE PHASE

Process Mapping (As-Is Process)

Purpose of As-Is Process Maps

- Document the current state of the process in detail
- Identify waste, bottlenecks, and variation sources
- Establish baseline for improvement

- Create common understanding among team members

Types of Process Maps

Flowchart:

- Basic process flow with decision points
- Uses standard symbols (rectangles for steps, diamonds for decisions)
- Good for simple processes

Swimlane Diagram:

- Shows which department/person performs each step
- Reveals handoffs and communication gaps
- Highlights accountability

Value Stream Map:

- Includes time, inventory, and information flow
- Distinguishes value-added from non-value-added activities
- Shows cycle time and lead time

Process Mapping Steps

1. Define process boundaries (start and end points)
2. Identify all process steps in sequence
3. Gather data on each step (time, resources, outputs)
4. Map the actual process, not the ideal process
5. Validate with process participants
6. Identify pain points and opportunities

Key Metrics to Capture

- **Cycle time**: Time to complete one unit
- **Lead time**: Total time from start to finish

- **Wait time**: Time spent waiting between steps
- **Rework rate**: Percentage requiring correction
- **Defect rate**: Errors per unit

11. Data Attributes (Continuous vs Discrete)

Continuous Data (Variable Data)

Definition: Data that can be measured on a continuous scale

Characteristics:

- Can take any value within a range
- Measured with instruments
- More informative than discrete data

Examples:

- Temperature (98.6°F, 99.2°F, 100.1°F)
- Weight (5.2 kg, 5.25 kg, 5.3 kg)
- Time (3.5 minutes, 4.2 minutes)
- Dimensions (length, width, height)
- Cost ($47.35, $52.18)

Statistical Tools Used:

- Mean, standard deviation
- Control charts (X-bar and R charts)
- Histograms
- Process capability indices (Cp, Cpk)

Discrete Data (Attribute Data)

Definition: Data that can be counted in whole numbers

Characteristics:

- Takes specific, distinct values
- Often yes/no or pass/fail
- Counted, not measured

Types:

Count Data:

- Number of defects per unit
- Examples: scratches on a surface, errors in a document

Classification Data:

- Pass/fail, good/bad, yes/no
- Examples: defective vs non-defective parts

Examples:

- Number of complaints (0, 1, 2, 3...)
- Defective items (5 defective out of 100)
- Customer ratings (1, 2, 3, 4, 5 stars)
- Number of accidents

Statistical Tools Used:

- Proportions, percentages
- Control charts (p, np, c, u charts)
- Pareto charts
- Chi-square tests

Choosing Between Continuous and Discrete

- **Prefer continuous data** when possible - provides more information
- Use discrete data when measurement is impractical or too expensive
- Sometimes convert continuous to discrete (e.g., temperature → fever/no fever)

12. Measurement System Analysis (MSA)

Purpose of MSA

- Ensure data is reliable and accurate
- Identify measurement errors before collecting data
- Determine if measurement system is capable

Components of Measurement Variation

Total Variation = Process Variation + Measurement Variation

Measurement Variation includes:

- **Repeatability (Equipment Variation)**: Same person, same item, multiple times
- **Reproducibility (Appraiser Variation)**: Different people, same item
- **Stability**: Consistency over time
- **Linearity**: Accuracy across measurement range
- **Bias**: Difference between measured and true value

Gage R&R Study (Repeatability & Reproducibility)

When to Use:

- Before starting data collection
- When measurement accuracy is questioned
- For critical measurements

Study Design:

- Typically 3 operators
- Measure 10 parts
- 2-3 trials each
- Calculate variation components

Acceptance Criteria:

- **%GRR < 10%**: Excellent measurement system
- **%GRR 10-30%**: May be acceptable depending on application
- **%GRR > 30%**: Unacceptable - improve before proceeding

Actions if System is Inadequate:

- Calibrate equipment
- Train operators
- Improve measurement procedure
- Use better measurement tools
- Narrow specification limits may require better system

Attribute Agreement Analysis

For Discrete Data: Check if inspectors agree with:

- Each other (between inspectors)
- Themselves over time (within inspector)
- A known standard

Metrics:

- **Percent agreement**: How often inspectors agree
- **Kappa statistic**: Agreement adjusted for chance (>0.7 is good)

13. Data Collection Techniques

Types of Data Collection

Sampling:

- Collect data from a subset of population
- Faster and less expensive than 100% inspection
- Must be random and representative

Census:

- Collect data from entire population
- Used when population is small
- Required for critical applications

Sampling Methods

Random Sampling:

- Every item has equal chance of selection
- Eliminates bias
- Example: Computer-generated random selection

Stratified Sampling:

- Divide population into groups (strata)
- Sample from each group proportionally
- Example: Sample from each shift, each supplier

Systematic Sampling:

- Select every nth item
- Example: Every 10th part off production line
- Risk: May miss patterns that coincide with sampling interval

Convenience Sampling:

- Sample what's easily available

- Not recommended - introduces bias
- Only use for preliminary studies

Sample Size Considerations

- **Larger samples**: More accurate, more expensive
- **Smaller samples**: Less accurate, more economical
- Consider: confidence level needed, expected variation, cost constraints
- Use statistical formulas or tables to determine appropriate size

Data Collection Tools

Check Sheets:

- Simple forms for tallying data
- Real-time collection
- Can show patterns visually

Automated Data Collection:

- Sensors and monitors
- Database queries
- Reduces human error

Surveys and Questionnaires:

- For customer feedback
- Voice of customer data
- Ensure questions are clear and unbiased

14. Data Collection Plan

Elements of a Data Collection Plan

What to Measure:

- Specific CTQ characteristics
- Output (Y) variables
- Process (X) variables

Operational Definition:

- Clear, precise definition of what's being measured
- Include how to measure it
- Ensure everyone interprets the same way
- Example: "Delivery time = hours from order placement to customer receipt"

Where to Collect:

- Which process steps
- Which locations/departments
- Which products/services

When to Collect:

- Time period
- Frequency (hourly, daily, weekly)
- Sample timing (random vs scheduled)

How Much Data:

- Sample size
- Number of data points needed for statistical validity
- Balance between accuracy and resources

Who Will Collect:

- Responsible person for each measurement
- Training requirements
- Backup collectors

How to Record:

- Data collection forms/templates
- Electronic systems
- Format and units

Creating the Plan

1. Link back to project charter and CTQs
2. Define operational definitions
3. Verify measurement system capability (MSA)
4. Design data collection forms
5. Train data collectors
6. Pilot test the plan
7. Adjust as needed
8. Execute collection

Data Quality Checks

- Verify completeness (no missing values)
- Check for outliers and errors
- Validate against expected ranges
- Review for consistency

15. Understanding Variation

Sources of Variation

Common Cause (Random) Variation:

- Inherent in the process
- Affects all outcomes similarly

- Predictable in aggregate
- Examples: minor temperature fluctuations, slight material differences
- **Action**: Process redesign needed to reduce

Special Cause (Assignable) Variation:

- Not part of normal process
- Creates unpredictable results
- Can be identified and eliminated
- Examples: machine breakdown, untrained operator, bad batch of materials
- **Action**: Investigate and eliminate the specific cause

Control Charts for Variation

Purpose: Distinguish between common and special cause variation

Stable Process:

- Only common cause variation present
- Predictable
- Points stay within control limits
- No patterns visible

Unstable Process:

- Special causes present
- Unpredictable
- Points outside control limits or showing patterns
- Must stabilize before improving

Voice of the Process vs Voice of the Customer

Voice of the Process (VOP):

- What the process actually delivers

- Measured by process capability
- Based on current performance data

Voice of the Customer (VOC):

- What customers require
- Specification limits
- CTQ requirements

Gap Analysis: Compare VOP to VOC to determine improvement needs

16. Measuring Process Capability

What is Process Capability?

The ability of a process to meet customer specifications consistently

Specification Limits

- **Upper Specification Limit (USL)**: Maximum acceptable value
- **Lower Specification Limit (LSL)**: Minimum acceptable value
- **Target**: Ideal value (often midpoint between USL and LSL)
- Set by customer requirements, not the process

Control Limits vs Specification Limits

Control Limits:

- Voice of the Process
- Based on actual process data (typically ±3 sigma from mean)
- Indicate process stability

Specification Limits:

- Voice of the Customer
- Set by customer or design requirements

- Determine if output is acceptable

Key Point: A stable process can still produce defects if control limits are wider than specification limits

Process Capability Indices

Cp (Potential Capability):

- Assumes process is centered
- Formula: $Cp = (USL - LSL) / (6\sigma)$
- Ignores process centering

Cpk (Actual Capability):

- Accounts for process centering
- Formula: $Cpk = \min[(USL - \mu)/(3\sigma), (\mu - LSL)/(3\sigma)]$
- More realistic than Cp

Interpretation:

- **Cpk ≥ 2.0**: Six Sigma level (excellent)
- **Cpk ≥ 1.33**: Adequate for most processes
- **Cpk = 1.0**: Process uses full specification range
- **Cpk < 1.0**: Process incapable - produces defects

Pp and Ppk:

- Similar to Cp and Cpk
- Used for preliminary studies or short-term capability
- Based on overall standard deviation

When Process Capability is Poor

If Cp is good but Cpk is poor:

- Process is not centered

- Solution: Center the process (adjust the mean)

If both Cp and Cpk are poor:

- Too much variation
- Solution: Reduce variation (improve the process)

17. Calculating Process Sigma Level

What is Process Sigma Level?

A metric indicating process capability in terms of defects per million opportunities (DPMO)

Calculation Steps

Step 1: Define Defect Opportunity

- What could go wrong?
- Each chance for a defect is one opportunity
- Example: A form with 20 fields has 20 opportunities per form

Step 2: Collect Data

- Total units inspected
- Total defects found
- Total opportunities for defects

Step 3: Calculate Metrics

DPO (Defects Per Opportunity):

- Formula: DPO = Total Defects / (Units × Opportunities per Unit)

DPMO (Defects Per Million Opportunities):

- Formula: DPMO = DPO × 1,000,000

Step 4: Convert to Sigma Level

- Use conversion table or calculator
- Includes 1.5 sigma shift (industry standard adjustment)

Example Calculation

- Inspected: 500 forms
- Defects found: 75
- Opportunities per form: 20

DPO = 75 / (500 × 20) = 0.0075 DPMO = 0.0075 × 1,000,000 = 7,500 Sigma Level ≈ 4.0 (from conversion table)

Sigma Level Benchmarks

- **2σ**: 308,537 DPMO (69% yield)
- **3σ**: 66,807 DPMO (93.3% yield)
- **4σ**: 6,210 DPMO (99.4% yield)
- **5σ**: 233 DPMO (99.977% yield)
- **6σ**: 3.4 DPMO (99.99966% yield)

The 1.5 Sigma Shift

- Accounts for long-term process drift
- Industry standard adjustment
- Short-term capability typically 1.5 sigma better than long-term

18. Visually Displaying Baseline Performance

Histogram

Purpose: Show distribution of continuous data

What to Look For:

- Central tendency (where data clusters)

- Spread (variation)
- Shape (normal, skewed, bimodal)
- Outliers

Interpretation:

- Bell-shaped: Normal distribution
- Skewed right: Tail extends right
- Bimodal: Two peaks (may indicate two different processes)

Box Plot (Box-and-Whisker Plot)

Shows:

- Median (middle line in box)
- Quartiles (box boundaries - 25th and 75th percentiles)
- Range (whiskers)
- Outliers (points beyond whiskers)

Advantages:

- Compare multiple groups easily
- Identify outliers quickly
- See spread at a glance

Run Chart

Purpose: Show data over time

Elements:

- Time on x-axis
- Measurement on y-axis
- Median line

Patterns to Detect:

- Trends (consistent increase/decrease)
- Shifts (sudden change in level)
- Cycles (repeating patterns)

Control Chart

Purpose: Distinguish common from special cause variation

Components:

- Center line (process mean)
- Upper Control Limit (UCL = mean + 3σ)
- Lower Control Limit (LCL = mean - 3σ)
- Data points over time

Rules for Special Causes:

- Point beyond control limits
- 8+ consecutive points on one side of center
- 6+ points trending up or down
- 14+ points alternating up and down

Scatter Diagram

Purpose: Show relationship between two variables

Interpretation:

- Positive correlation: Both increase together
- Negative correlation: One increases as other decreases
- No correlation: Random scatter
- Strength indicated by how tightly points cluster

Pareto Chart

Purpose: Identify the vital few from the trivial many (80/20 rule)

Elements:

- Bars showing frequency or impact (descending order)
- Line showing cumulative percentage

Use: Focus improvement efforts on biggest contributors

19. Statistics Fundamentals

Descriptive Statistics

Measures of Central Tendency:

Mean (Average):

- Formula: Sum of all values / Number of values
- Sensitive to outliers
- Most commonly used

Median:

- Middle value when data is sorted
- Not affected by outliers
- Better for skewed distributions

Mode:

- Most frequently occurring value
- Can have multiple modes
- Useful for categorical data

Measures of Spread:

Range:

- Maximum value - Minimum value
- Simple but affected by outliers

Standard Deviation (σ or s):

- Average distance from the mean
- Most important measure of variation
- Formula involves squared differences from mean
- Population: σ, Sample: s

Variance:

- Standard deviation squared
- Used in many statistical calculations

Interquartile Range (IQR):

- Range of middle 50% of data (Q3 - Q1)
- Not affected by outliers

Normal Distribution

Characteristics:

- Bell-shaped, symmetrical
- Mean = Median = Mode
- Defined by mean (μ) and standard deviation (σ)

Empirical Rule (68-95-99.7):

- 68% of data within $\pm 1\sigma$ from mean
- 95% within $\pm 2\sigma$
- 99.7% within $\pm 3\sigma$

Importance: Many statistical tools assume normal distribution

Testing for Normality

Visual Methods:

- Histogram (should be bell-shaped)

- Normal probability plot (points should follow straight line)

Statistical Tests:

- Anderson-Darling test
- Shapiro-Wilk test
- p-value > 0.05 suggests normality

What if Data is Not Normal?:

- Transform data (log, square root)
- Use non-parametric tests
- Collect more data
- Check for special causes

20. Probability and Probability Distributions

Basic Probability

Probability: Likelihood of an event occurring (0 to 1, or 0% to 100%)

Formula: P(event) = Number of favorable outcomes / Total possible outcomes

Rules:

- Addition rule: P(A or B) = P(A) + P(B) - P(A and B)
- Multiplication rule: P(A and B) = P(A) × P(B) if independent
- Complement rule: P(not A) = 1 - P(A)

Probability Distributions

Normal Distribution:

- Continuous data
- Bell-shaped curve
- Defined by μ (mean) and σ (standard deviation)
- Used for many natural phenomena

Binomial Distribution:

- Discrete data
- Fixed number of trials
- Two possible outcomes (success/failure)
- Example: Number of defective items in a sample

Poisson Distribution:

- Discrete data
- Counts events in a fixed interval
- Example: Number of defects per unit
- Useful when probability of defect is small

Exponential Distribution:

- Continuous data
- Time between events
- Example: Time between machine failures
- Used in reliability analysis

Z-Score (Standard Score)

Formula: $Z = (X - \mu) / \sigma$

Interpretation:

- Number of standard deviations from the mean
- Z = 0: Value equals the mean
- Z = 1: Value is 1 standard deviation above mean
- Z = -2: Value is 2 standard deviations below mean

Use: Calculate probability of a value occurring

Example:

- Mean = 100, σ = 10, X = 115
- Z = (115 - 100) / 10 = 1.5
- Look up Z = 1.5 in table: 93.3% of values are below 115

21. Process Performance Analysis

Short-term vs Long-term Performance

Short-term (Within subgroup):

- Minimal variation
- Process under control
- Used for Cp, Cpk calculations

Long-term (Overall):

- Includes all sources of variation
- More realistic
- Used for Pp, Ppk calculations

Performance Gap: Difference between short-term and long-term capability indicates room for improvement

Yield Metrics

First Time Yield (FTY):

- Percentage of units passing first time, no rework
- Formula: (Units passing) / (Total units) × 100%

Rolled Throughput Yield (RTY):

- Probability of passing all process steps
- Formula: $FTY_1 \times FTY_2 \times FTY_3 \times ... \times FTY_n$
- Shows cumulative effect of defects

Example:

- Step 1: 95% FTY
- Step 2: 98% FTY
- Step 3: 97% FTY
- RTY = 0.95 × 0.98 × 0.97 = 0.903 = 90.3%

Defects Per Unit (DPU)

Formula: DPU = Total defects / Total units

Relationship to Yield: Yield = e^(-DPU)

Hidden Factory:

- Work required to fix defects
- Not visible in output but consumes resources
- Calculated from rework and scrap rates

Benchmarking

Internal Benchmarking:

- Compare across different locations, shifts, products
- Identify best practices

External Benchmarking:

- Compare to competitors or industry leaders
- Understand competitive position

Metrics to Compare:

- Sigma levels
- Cycle times
- Defect rates
- Customer satisfaction

- Cost per unit

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Elements:

- Bars showing frequency or impact (descending order)
- Line showing cumulative percentage

Use: Focus improvement efforts on biggest contributors

19. Statistics Fundamentals

Descriptive Statistics

Measures of Central Tendency:

Mean (Average):

- Formula: Sum of all values / Number of values
- Sensitive to outliers
- Most commonly used

Median:

- Middle value when data is sorted
- Not affected by outliers
- Better for skewed distributions

Mode:

- Most frequently occurring value
- Can have multiple modes
- Useful for categorical data

Measures of Spread:

Range:

- Maximum value - Minimum value
- Simple but affected by outliers

Standard Deviation (σ or s):

- Average distance from the mean
- Most important measure of variation
- Formula involves squared differences from mean
- Population: σ, Sample: s

Variance:

- Standard deviation squared
- Used in many statistical calculations

Interquartile Range (IQR):

- Range of middle 50% of data (Q3 - Q1)
- Not affected by outliers

Normal Distribution

Characteristics:

- Bell-shaped, symmetrical
- Mean = Median = Mode
- Defined by mean (μ) and standard deviation (σ)

Empirical Rule (68-95-99.7):

- 68% of data within $\pm 1\sigma$ from mean
- 95% within $\pm 2\sigma$
- 99.7% within $\pm 3\sigma$

Importance: Many statistical tools assume normal distribution

Testing for Normality

Visual Methods:

- Histogram (should be bell-shaped)
- Normal probability plot (points should follow straight line)

Statistical Tests:

- Anderson-Darling test
- Shapiro-Wilk test
- p-value > 0.05 suggests normality

What if Data is Not Normal?:

- Transform data (log, square root)
- Use non-parametric tests
- Collect more data
- Check for special causes

20. Probability and Probability Distributions

Basic Probability

Probability: Likelihood of an event occurring (0 to 1, or 0% to 100%)

Formula: P(event) = Number of favorable outcomes / Total possible outcomes

Rules:

- Addition rule: P(A or B) = P(A) + P(B) - P(A and B)

- Multiplication rule: P(A and B) = P(A) × P(B) if independent
- Complement rule: P(not A) = 1 - P(A)

Probability Distributions

Normal Distribution:

- Continuous data
- Bell-shaped curve
- Defined by μ (mean) and σ (standard deviation)
- Used for many natural phenomena

Binomial Distribution:

- Discrete data
- Fixed number of trials
- Two possible outcomes (success/failure)
- Example: Number of defective items in a sample

Poisson Distribution:

- Discrete data
- Counts events in a fixed interval
- Example: Number of defects per unit
- Useful when probability of defect is small

Exponential Distribution:

- Continuous data
- Time between events
- Example: Time between machine failures
- Used in reliability analysis

Z-Score (Standard Score)

Formula: $Z = (X - \mu) / \sigma$

Interpretation:

- Number of standard deviations from the mean
- Z = 0: Value equals the mean
- Z = 1: Value is 1 standard deviation above mean
- Z = -2: Value is 2 standard deviations below mean

Use: Calculate probability of a value occurring

Example:

- Mean = 100, σ = 10, X = 115
- Z = (115 - 100) / 10 = 1.5
- Look up Z = 1.5 in table: 93.3% of values are below 115

21. Process Performance Analysis

Short-term vs Long-term Performance

Short-term (Within subgroup):

- Minimal variation
- Process under control
- Used for Cp, Cpk calculations

Long-term (Overall):

- Includes all sources of variation
- More realistic
- Used for Pp, Ppk calculations

Performance Gap: Difference between short-term and long-term capability indicates room for improvement

Yield Metrics

First Time Yield (FTY):

- Percentage of units passing first time, no rework
- Formula: (Units passing) / (Total units) × 100%

Rolled Throughput Yield (RTY):

- Probability of passing all process steps
- Formula: $FTY_1 \times FTY_2 \times FTY_3 \times ... \times FTY_n$
- Shows cumulative effect of defects

Example:

- Step 1: 95% FTY
- Step 2: 98% FTY
- Step 3: 97% FTY
- RTY = 0.95 × 0.98 × 0.97 = 0.903 = 90.3%

Defects Per Unit (DPU)

Formula: DPU = Total defects / Total units

Relationship to Yield: Yield = $e^{(-DPU)}$

Hidden Factory:

- Work required to fix defects
- Not visible in output but consumes resources
- Calculated from rework and scrap rates

Benchmarking

Internal Benchmarking:

- Compare across different locations, shifts, products
- Identify best practices

External Benchmarking:

- Compare to competitors or industry leaders
- Understand competitive position

Metrics to Compare:

- Sigma levels
- Cycle times
- Defect rates
- Customer satisfaction
- Cost per unit

Statistical Process Control (SPC)

Overview

Statistical Process Control is a method of quality control that uses statistical techniques to monitor and control a process, ensuring it operates at its full potential.

Key Concepts

- **Process Variation**: Natural (common cause) vs. assignable (special cause) variation
- **Control Limits**: Upper Control Limit (UCL) and Lower Control Limit (LCL)
- **Process Capability**: Measuring how well a process meets specifications
- **Rational Subgroups**: Samples taken to represent process performance

Objectives of SPC

- Reduce variability in processes
- Detect problems early before defects occur
- Improve process capability and performance
- Make data-driven decisions

- Achieve consistent quality output

2. Variable Control Charts

Variable data involves measurements on a continuous scale (length, weight, temperature, time).

Common Variable Control Charts

X-bar and R Chart (Mean and Range)

- **X-bar chart**: Monitors process mean
- **R chart**: Monitors process variability
- Used for small sample sizes ($n \leq 10$)
- Formulas:
  - X-bar = $\Sigma x / n$
  - R = Max - Min
  - UCL(X-bar) = $\bar{\bar{X}} + A_2\bar{R}$
  - LCL(X-bar) = $\bar{\bar{X}} - A_2\bar{R}$

X-bar and S Chart (Mean and Standard Deviation)

- **S chart**: Uses standard deviation instead of range
- Better for larger sample sizes ($n > 10$)
- More sensitive to process changes

Individual and Moving Range (I-MR) Chart

- Used when only one measurement per subgroup
- Common in chemical processes, batch operations

- Moving range = $|X_i - X_{i-1}|$

Control Limit Constants

- A_2, D_3, D_4 for R charts
- A_3, B_3, B_4 for S charts
- Values depend on sample size (n)

3. Attribute Control Charts

Attribute data represents countable characteristics (defective/non-defective, pass/fail).

Types of Attribute Control Charts

p-Chart (Proportion Defective)

- Monitors proportion of defective units
- Variable sample size possible
- p = number of defectives / sample size
- UCL = $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$
- LCL = $\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$

np-Chart (Number Defective)

- Monitors number of defective units
- Requires constant sample size
- np = number of defectives
- UCL = $n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

c-Chart (Number of Defects)

- Monitors count of defects per unit
- Used when defects per unit can exceed one
- Based on Poisson distribution

- UCL = $\bar{c} + 3\sqrt{\bar{c}}$
- LCL = $\bar{c} - 3\sqrt{\bar{c}}$

u-Chart (Defects per Unit)

- Monitors average defects per unit
- Variable sample size allowed
- u = total defects / total units
- UCL = $\bar{u} + 3\sqrt{(\bar{u}/n)}$

4. Operating Characteristic (OC) Curve

Definition

A graph showing the probability of accepting a lot versus the actual quality level of the lot.

Key Elements

- **X-axis**: Actual proportion defective (p or fraction defective)
- **Y-axis**: Probability of acceptance (Pa)
- **Ideal OC Curve**: Vertical line at Acceptable Quality Level (AQL)
- **Real OC Curve**: S-shaped curve

Important Points on OC Curve

- **Producer's Risk (α)**: Probability of rejecting good lots (Type I error)
- **Consumer's Risk (β)**: Probability of accepting bad lots (Type II error)
- **AQL**: Acceptable Quality Level
- **LTPD/RQL**: Lot Tolerance Percent Defective / Rejectable Quality Level

Applications

- Evaluate sampling plans

- Compare different inspection strategies
- Balance producer and consumer risks
- Determine appropriate sample sizes

5. Acceptance Sampling

Overview

Inspection of a sample from a lot to decide acceptance or rejection of the entire lot.

Types of Sampling Plans

Single Sampling Plan

- Take one sample of size n
- Accept if defectives ≤ c (acceptance number)
- Reject if defectives > c

Double Sampling Plan

- First sample: Accept if $d \leq c_1$, reject if $d > c_2$
- Second sample taken if $c_1 < d \leq c_2$
- More economical on average

Multiple Sampling Plan

- Series of small samples
- Decision after each sample
- Most complex but efficient

Sequential Sampling

- Items inspected one at a time
- Decision made after each inspection
- Minimum sample size on average

Key Parameters

- **n**: Sample size
- **c**: Acceptance number
- **AQL**: Acceptable Quality Level (typically 1-4%)
- **LTPD**: Lot Tolerance Percent Defective
- **AOQ**: Average Outgoing Quality
- **AOQL**: Average Outgoing Quality Limit

Sampling Standards

- **MIL-STD-105E**: Military standard (now ANSI/ASQ Z1.4)
- **Dodge-Romig Tables**: Minimize inspection costs

6. Minitab Applications

Overview

Minitab is statistical software widely used for quality control and Six Sigma projects.

Key SPC Functions in Minitab

Creating Control Charts

- **Stat > Control Charts > Variables Charts**: X-bar, R, S, I-MR charts
- **Stat > Control Charts > Attributes Charts**: p, np, c, u charts
- Automatic calculation of control limits
- Visual identification of out-of-control points

Process Capability Analysis

- **Stat > Quality Tools > Capability Analysis**
- Calculates Cp, Cpk, Pp, Ppk indices
- Normal, non-normal, and attribute capability

- Visual histograms with specification limits

Acceptance Sampling Plans

- **Stat > Quality Tools > Acceptance Sampling**
- Create and compare sampling plans
- Generate OC curves
- Calculate AOQ and ATI

Pareto Charts

- Identify major contributors to problems
- 80-20 rule visualization

Gage R&R Studies

- Evaluate measurement system variation
- Repeatability and reproducibility analysis

Common Outputs

- Control charts with interpretation
- Process capability reports
- Statistical summaries
- Hypothesis test results

7. Design for Six Sigma (DFSS)

Overview

DFSS is a systematic methodology for designing products and processes to meet Six Sigma quality levels (3.4 defects per million opportunities).

Objectives

- Design quality into products from inception

- Prevent defects rather than detect them
- Reduce development time and costs
- Achieve customer satisfaction
- Optimize design for manufacturability

Key Principles

- Voice of Customer (VOC) driven
- Robust design against variation
- Predictive modeling and simulation
- Design verification and validation
- Risk assessment and mitigation

8. DMADV Methodology

DMADV is a Six Sigma framework for designing new products or processes.

Define Phase

- Identify project goals and customer requirements
- Define Voice of Customer (VOC)
- Create project charter
- Develop high-level process maps
- **Tools**: Project charter, stakeholder analysis, CTQ tree

Measure Phase

- Translate customer needs into measurable specifications
- Identify Critical to Quality (CTQ) characteristics
- Benchmark competitors
- Conduct market research

- **Tools**: QFD (Quality Function Deployment), Kano model, surveys

Analyze Phase

- Develop design alternatives
- Perform concept selection
- Conduct trade-off analysis
- Use predictive models
- **Tools**: Pugh matrix, FMEA, DOE (Design of Experiments)

Design Phase

- Develop detailed design
- Optimize design parameters
- Create prototypes
- Test and validate design
- **Tools**: CAD/CAM, simulation, tolerance analysis, DOE

Verify Phase

- Pilot runs and testing
- Validate design meets requirements
- Implement process controls
- Transfer to production
- **Tools**: Control plans, process capability studies, validation testing

9. DMADOV Methodology

DMADOV extends DMADV by adding an Optimize phase.

Structure

- **Define**: Same as DMADV

- **Measure**: Same as DMADV
- **Analyze**: Same as DMADV
- **Design**: Initial design development
- **Optimize**: Refine and optimize the design
- **Verify**: Validate final design

Optimize Phase (Additional)

- Fine-tune design parameters
- Reduce variation through robust design
- Optimize cost-performance balance
- Use advanced DOE techniques
- Taguchi methods for robustness
- **Tools**: Response surface methodology, Monte Carlo simulation

When to Use DMADOV

- Complex designs requiring optimization
- When multiple competing objectives exist
- Products with high variability concerns
- Cost-sensitive applications
- Projects requiring robust design

10. DFX (Design for X)

Overview

DFX represents a collection of design methodologies where "X" stands for various attributes.

Common DFX Approaches

Design for Manufacturability (DFM)

- Simplify product design
- Reduce part count
- Standardize components
- Minimize assembly complexity
- Consider production capabilities

Design for Assembly (DFA)

- Minimize assembly operations
- Self-locating features
- Easy part handling
- Reduce fasteners
- Error-proofing (poka-yoke)

Design for Reliability (DFR)

- Predict failure modes
- Stress testing and analysis
- Redundancy where needed
- Derating components
- FMEA integration

Design for Testability (DFT)

- Built-in test points
- Easy access for inspection
- Diagnostic features
- Modular design for isolation

Design for Cost (DFC)

- Target costing
- Value engineering
- Material selection
- Process optimization
- Total cost of ownership

Design for Environment (DFE)

- Eco-friendly materials
- Energy efficiency
- Recyclability
- Minimal waste
- Sustainable sourcing

Design for Service/Maintainability (DFS)

- Easy disassembly
- Accessible components
- Modular replacement
- Clear documentation
- Diagnostic aids

Integration with DFSS

- DFX principles applied throughout DMADV/DMADOV
- Concurrent consideration of multiple X factors
- Trade-off analysis among different DFX goals
- Use of scoring matrices and decision tools
- Cross-functional team collaboration